

2 | POLYNOMIALS

EXERCISE 2.1

Q.1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$ (iii) $3\sqrt{t} + t\sqrt{2}$ (iv) $y + \frac{2}{y}$ (v) $x^{10} + y^3 + t^{50}$

Sol. (i) Polynomial in one variable, x **Ans.**

(ii) Polynomial in one variable, y . **Ans.**

(iii) $3\sqrt{t} + t\sqrt{2}$ is not a polynomial as power of t in \sqrt{t} is not a whole number. **Ans.**

(iv) $y + \frac{2}{y}$ is not a polynomial as power of y in second term, i.e., $\frac{1}{y} = y^{-1}$ is not a whole number. **Ans.**

(v) $x^{10} + y^3 + t^{50}$ is not a polynomial in one variable but a polynomial in three variables x , y and t . **Ans.**

Q.2. Write the coefficients of x^2 in each of the following :

(i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$ (iii) $\frac{\pi}{2}x^2 + x$ (iv) $\sqrt{2}x - 1$

Sol. (i) In $2 + x^2 + x$, coefficient of x^2 is 1. **Ans.**

(ii) In $2 - x^2 + x^3$, coefficient of x^2 is -1 . **Ans.**

(iii) $\frac{\pi}{2}x^2 + x$, coefficient of x^2 is $\frac{\pi}{2}$. **Ans.**

(iv) $\sqrt{2}x - 1$, x^2 is not present hence no coefficient. **Ans.**

Q.3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Sol. $x^{35} + 5$ is a binomial of degree 35.

$2y^{100}$ is a monomial of degree 100. **Ans.**

Q.4. Write the degree of each of the following polynomials :

(i) $5x^3 + 4x^2 + 7x$ (ii) $4 - y^2$ (iii) $5t - \sqrt{7}$ (iv) 3

Sol. (i) Degree is 3 as x^3 is the highest power. **Ans.**

(ii) Degree is 2 as y^2 is the highest power. **Ans.**

(iii) Degree is 1 as t is the highest power. **Ans.**

(iv) Degree is 0 as x^0 is the highest power. **Ans.**

Q.5. Classify the following as linear, quadratic and cubic polynomials :

(i) $x^2 + x$ (ii) $x - x^3$ (iii) $y + y^2 + 4$

(iv) $1 + x$ (v) $3t$ (vi) r^2 (vii) $7x^3$

Sol. (i) $x^2 + x$ is quadratic. (ii) $x - x^3$ is cubic.

(iii) $y + y^2 + 4$ is quadratic. (iv) $1 + x$ is linear.

(v) $3t$ is linear.

(vi) r^2 is quadratic.

(vii) $7x^3$ is cubic.

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EXERCISE 2.2

Q.1. Find the value of the polynomial $5x - 4x^2 + 3$ at

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Sol. $p(x) = 5x - 4x^2 + 3$

(i) At $x = 0$, $p(0) = 5 \times 0 - 4 \times 0^2 + 3 = 3$ **Ans.**

(ii) At $x = -1$, $p(-1) = 5 \times (-1) - 4 \times (-1)^2 + 3 = -5 - 4 + 3 = -6$ **Ans.**

(iii) At $x = 2$, $p(2) = 5 \times 2 - 4 \times (2)^2 + 3 = 10 - 16 + 3 = -3$ **Ans.**

Q.2. Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials :

(i) $p(y) = y^2 - y + 1$ (ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$ (iv) $p(x) = (x - 1)(x + 1)$

Sol. (i) $p(y) = y^2 - y + 1$

$p(0) = 0^2 - 0 + 1 = 1$

$p(1) = 1^2 - 1 + 1 = 1$

$p(2) = 2^2 - 2 + 1 = 3$. **Ans.**

(ii) $p(t) = 2 + t + 2t^2 - t^3$

$p(0) = 2 + 0 + 2 \times 0^2 - 0^3 = 2$

$p(1) = 2 + 1 + 2 \times 1^2 - 1^3 = 4$

$p(2) = 2 + 2 + 2 \times 2^2 - 2^3 = 4 + 8 - 8 = 4$.

(iii) $p(x) = x^3$

$p(0) = 0$

$p(1) = 1$

$p(2) = 8$. **Ans.**

(iv) $p(x) = (x - 1)(x + 1)$

$p(0) = (-1)(1) = -1$

$p(1) = (1 - 1)(1 + 1) = 0$

$p(2) = (2 - 1)(2 + 1) = 3$ **Ans.**

Q.3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1$, $x = -\frac{1}{3}$ (ii) $p(x) = 5x - \pi$, $x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1$, $x = 1, -1$

(iv) $p(x) = (x + 1)(x - 2)$, $x = -1, 2$

(v) $p(x) = x^2$, $x = 0$ (vi) $p(x) = lx + m$, $x = -\frac{m}{l}$

(vii) $p(x) = 3x^2 - 1$, $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x) = 2x + 1$, $x = \frac{1}{2}$

Sol. (i) Yes. $3x + 1 = 0$, for $x = -\frac{1}{3}$. **Ans.**

(ii) No. $5x - \pi = 5 \times \frac{4}{5} - \pi = 4 - \pi \neq 0$ **Ans.**

(iii) Yes. $x^2 - 1 = 1^2 - 1 = 0$ for $x = 1$

and $x^2 - 1 = (-1)^2 - 1 = 0$ for $x = -1$ **Ans.**

(iv) Yes. $(x + 1)(x - 2) = 0$ for $x = -1$, or, $x = 2$.

(v) Yes. $x^2 = 0$ for $x = 0$

(vi) Yes. $lx + m = 0$ for $x = -\frac{m}{l}$

(vii) $3x^2 - 1 = 3\frac{1}{3} - 1 = 0$ for $x = \frac{-1}{\sqrt{3}}$

and $3x^2 - 1 = 3 \cdot \frac{4}{3} - 1 = 3 \neq 0$

Thus, for $\frac{-1}{\sqrt{3}}$ is a zero but $-\frac{2}{\sqrt{3}}$ is not a zero of the polynomial **Ans.**

(viii) No. $2x + 1 \neq 0$ for $x = \frac{1}{2}$.

Q.4. Find the zero of the polynomial in each of the following cases :

(i) $p(x) = x + 5$

(ii) $p(x) = x - 5$

(iii) $p(x) = 2x + 5$

(iv) $p(x) = 3x - 2$

(v) $p(x) = 3x$

(vi) $p(x) = ax, a \neq 0$

(vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.

Sol. (i) $x + 5 = 0, x = -5$, so, -5 is the zero of $x + 5$ **Ans.**

(ii) $x - 5 = 0, x = 5$ so, 5 is the zero of $x - 5$ **Ans.**

(iii) $2x + 5 = 0, \Rightarrow 2x = -5,$

$\Rightarrow x = \frac{-5}{2}$, so $-\frac{5}{2}$ is the zero of $2x + 5$ **Ans.**

(iv) $3x - 2 = 0 \Rightarrow 3x = 2$

$\Rightarrow x = \frac{2}{3}$, so $\frac{2}{3}$ is the zero of $3x - 2$ **Ans.**

(v) $3x = 0, \Rightarrow x = 0$, so 0 is the zero of $3x$ **Ans.**

(vi) $ax = 0 (a \neq 0) \Rightarrow x = \frac{0}{a} = 0$, so, 0 is the zero of ax **Ans.**

(vii) $cx + d = 0 (c \neq 0)$

$\Rightarrow cx = -d$

$\Rightarrow x = \frac{-d}{c}$, so, $\frac{-d}{c}$ is the 0 of $cx + d$ **Ans.**

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EXERCISE 2.3

Q.1. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$ (ii) $x - \frac{1}{2}$ (iii) x

(iv) $x + \pi$ (v) $5 + 2x$

Sol. $p(x) = x^3 + 3x^2 + 3x + 1$

(i) When $p(x)$ is divided by $x + 1$,

i.e., $x + 1 = 0$, $x = -1$ is to be substituted in $p(x)$.

$$\begin{aligned} p(-1) &= (-1)^3 + 3(-1)^2 + 3(-1) + 1 \\ &= -1 + 3 - 3 + 1 = 0 \end{aligned}$$

Remainder = 0. **Ans.**

(ii) When $p(x)$ is divided by $x - \frac{1}{2}$ remainder is $p\left(\frac{1}{2}\right)$.

$$\begin{aligned} p\left(\frac{1}{2}\right) &= \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1 \\ &= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 = \frac{1+6+12+8}{8} \end{aligned}$$

$$\therefore \text{Remainder} = \frac{27}{8} = 3\frac{3}{8} \text{ Ans.}$$

(iii) When $p(x)$ is divided by x , then remainder is $p(0)$.

$x = 0$, substitute in $p(x)$

$$p(0) = 0^3 + 3 \times 0^2 + 3 \times 0 + 1 = 1.$$

\therefore Remainder = 1 **Ans.**

(iv) When $p(x)$ is divided by $x + \pi$, then, remainder is $p(-\pi)$.

$x = -\pi$ to be substituted in $p(x)$

$$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1.$$

\therefore Remainder = $-\pi^3 + 3\pi^2 - 3\pi + 1$ **Ans.**

(v) When $p(x)$ is divided by $(5 + 2x)$, then remainder is $p\left(\frac{-5}{2}\right)$.

$$\begin{aligned} p\left(\frac{-5}{2}\right) &= \left(\frac{-5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 1 \\ &= \frac{-125}{8} + \frac{75}{4} - \frac{15}{2} + 1 = \frac{-125 + 150 - 60 + 8}{8} \end{aligned}$$

$$\text{Remainder} = \frac{-35 + 8}{8} = \frac{-27}{8} \text{ Ans.}$$

Q.2. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Sol. $p(x) = x^3 - ax^2 + 6x - a$

When $p(x)$ is divided by $x - a$, the remainder is $p(a)$.

Substitute $x = a$ in $p(x)$

$$p(a) = a^3 - a^3 + 6a - a = 5a \quad \mathbf{Ans.}$$

Q.3. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Sol. $7 + 3x = 0$

$$\Rightarrow 3x = -7$$

$$\Rightarrow x = \frac{-7}{3}$$

Substitute $x = \frac{-7}{3}$ in $p(x) = 3x^3 + 7x$

$$p\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right) = \frac{-343}{9} - \frac{49}{3} = \frac{-343 - 147}{9} = \frac{-490}{9}.$$

So, remainder = $\frac{-490}{9}$ which is different from 0.

Therefore, $(3x + 7)$ is not a factor of the polynomial $3x^3 + 7x$. **Ans.**

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EXERCISE 2.4

Q.1. Determine which of the following polynomials has $(x + 1)$ a factor :

- (i) $x^3 + x^2 + x + 1$ (ii) $x^4 + x^3 + x^2 + x + 1$
 (iii) $x^4 + 3x^3 + 3x^2 + x + 1$ (iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Sol. To have $(x + 1)$ as a factor, substituting $x = -1$ must give $p(-1) = 0$.

(i) $x^3 + x^2 + x + 1$
 $= (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$

Therefore, $x + 1$ is a factor of $x^3 + x^2 + x + 1$ **Ans.**

(ii) $x^4 + x^3 + x^2 + x + 1$
 $= (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 = 1 - 1 + 1 - 1 + 1 = 1$

Remainder is not 0. Therefore $(x + 1)$ is not its factor. **Ans.**

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$
 $= (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$
 $= 1 - 3 + 3 - 1 + 1 = 1$. Remainder is not 0

Therefore, $(x + 1)$ is not its factor. **Ans.**

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$
 $= (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$
 $= -1 - 1 + 2 + \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

Remainder not 0, therefore $(x + 1)$ is not a factor. **Ans.**

Q.2. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases :

- (i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$
 (ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$
 (iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

Sol. (i) $g(x) = x + 1$. $x = -1$ to be substituted in

$$p(x) = 2x^3 + x^2 - 2x - 1$$

$$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1 = -2 + 1 + 2 - 1 = 0.$$

So, $g(x)$ is a factor of $p(x)$. **Ans.**

(ii) $g(x) = x + 2$, substitute $x = -2$ in $p(x)$

$$p(x) = x^3 + 3x^2 + 3x + 1$$

$$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1 = -8 + 12 - 6 + 1 = -1.$$

So, $g(x)$ is not a factor of $p(x)$ **Ans.**

(iii) $g(x) = x - 3$ substitute $x = 3$ in $p(x)$.

$$p(x) = x^3 - 4x^2 + x + 6$$

$$p(3) = (3)^3 - 4(3)^2 + 3 + 6 = 27 - 36 + 3 + 6 = 0.$$

Therefore, $g(x)$ is a factor of $x^3 - 4x^2 + x + 6$. **Ans.**

Q.3. Find the value of k , if $x - 1$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = x^2 + x + k$ (ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$ (iv) $p(x) = kx^2 - 3x + k$

Sol. $(x - 1)$ is a factor, so we substitute $x = 1$ in each case and solve for k by making $p(1)$ equal to 0.

(i) $p(x) = x^2 + x + k$
 $p(1) = 1 + 1 + k = 0 \Rightarrow k = -2$ **Ans.**

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$
 $p(1) = 2 \times 1^2 + k \times 1 + \sqrt{2} = 0$
 $\Rightarrow 2 + k + \sqrt{2} = 0$
 $\Rightarrow k = -2 - \sqrt{2} = -(2 + \sqrt{2})$ **Ans.**

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$
 $p(1) = k - \sqrt{2} + 1 = 0$
 $\Rightarrow k = \sqrt{2} - 1$ **Ans.**

(iv) $p(x) = kx^2 - 3x + k$
 $p(1) = k - 3 + k = 0 \Rightarrow 2k - 3 = 0$
 $\Rightarrow k = \frac{3}{2}$ **Ans.**

Q.4. Factorise :

(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$ (iii) $6x^2 + 5x - 6$ (iv) $3x^2 - x - 4$

Sol. (i) $12x^2 - 7x + 1$
 $= 12x^2 - 4x - 3x + 1$
 $= 4x(3x - 1) - 1(3x - 1) = (4x - 1)(3x - 1)$ **Ans.**

(ii) $2x^2 + 7x + 3$
 $= 2x^2 + 6x + x + 3$
 $= 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$ **Ans.**

(iii) $6x^2 + 5x - 6$
 $= 6x^2 + 9x - 4x - 6$
 $= 3x(2x + 3) - 2(2x + 3) = (3x - 2)(2x + 3)$ **Ans.**

(iv) $3x^2 - x - 4$
 $= 3x^2 - 4x + 3x - 4 = x(3x - 4) + 1(3x - 4) = (x + 1)(3x - 4)$ **Ans.**

Q.5. Factorise :

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$ (iv) $2y^3 + y^2 - 2y - 1$

Sol. (i) $p(x) = x^3 - 2x^2 - x + 2$

Let us guess a factor $(x - a)$ and choose value of a arbitrarily as 1.
 Now, putting this value in $p(x)$.

$$1 - 2 - 1 + 2 = 0$$

So $(x - 1)$ is a factor of $p(x)$

$$\begin{aligned} \text{Now, } x^3 - 2x^2 - x + 2 &= x^3 - x^2 - x^2 + x - 2x + 2 \\ &= x^2(x - 1) - x(x - 1) - 2(x - 1) \\ &= (x - 1)(x^2 - x - 2) \\ &= (x - 1)(x^2 - 2x + x - 2) \\ &= (x - 1)\{x(x - 2) + 1(x - 2)\} \\ &= (x - 1)(x + 1)(x - 2) \text{ **Ans.**} \end{aligned}$$

To factorise it

$$x^2 - 2x + x - 2 = x(x - 2) + 1(x - 2) = (x + 1)(x - 2).$$

After factorisation $(x - 1)(x + 1)(x - 2)$.

(ii) $p(x) = x^3 - 3x^2 - 9x - 5$

Take a factor $(x - a)$. a should be a factor of 5, i.e., ± 1 or ± 5 .

For $(x - 1)$, $a = 1$

$$\begin{aligned} p(1) &= (1)^3 - (-3) 1^2 - 9 \times 1 - 5 \\ &= 1 - 3 - 9 - 5 = -16. \end{aligned}$$

So, $(x - 1)$ is not a factor of $p(x)$.

For $a = 5$

$$\begin{aligned} p(5) &= (5)^3 - 3(5)^2 - 9(5) - 5 \\ &= 125 - 75 - 45 - 5 = 0. \end{aligned}$$

Therefore, $(x - 5)$ is a factor of $x^3 - 3x^2 - 9x - 5$.

$$\begin{aligned} \text{Now, } x^3 - 3x^2 - 9x - 5 &= x^3 - 5x^2 + 2x^2 - 10x + x - 5 \\ &= x^2(x - 5) + 2x(x - 5) + 1(x - 5) \\ &= (x - 5)(x^2 + 2x + 1) \\ &= (x - 5)(x + 1)^2 \\ &= (x - 5)(x + 1)(x + 1) \end{aligned}$$

So, $x^3 - 3x^2 - 9x - 5 = (x - 5)(x + 1)(x + 1)$. **Ans.**

(iii) $p(x) = x^3 + 13x^2 + 32x + 20$

Let a factor be $(x - a)$. a should be a factor of 20 which are ± 1 , ± 2 , ± 4 , ± 5 , ± 10 .

For $x - 1 = 0 \Rightarrow x = 1$

$$\begin{aligned} \text{Now, } p(1) &= 1 + 13 + 32 + 20 \\ &= 66 \neq 0 \end{aligned}$$

Hence, $(x - 1)$ is not a factor of $p(x)$.

Again, for $x + 1 = 0 \Rightarrow x = -1$

$$\begin{aligned} \text{Now, } p(-1) &= -1 + 13 - 32 + 20 \\ &= -33 + 33 = 0 \end{aligned}$$

Hence, $(x + 1)$ is a factors of $p(x)$.

$$\begin{aligned} \text{Now, } x^3 + 13x^2 + 32x + 20 &= x^3 + x^2 + 12x^2 + 20x + 20 \\ &= x^2(x + 1) + 12x(x + 1) + 20(x + 1) \\ &= (x + 1)(x^2 + 12x + 20) \\ &= (x + 1)(x^2 + 10x + 2x + 20) \\ &= (x + 1)\{x(x + 10) + 2(x + 10)\} \\ &= (x + 2)(x + 1)(x + 10) \quad \mathbf{Ans.} \end{aligned}$$

(iv) $p(y) = 2y^3 + y^2 - 2y - 1$

factors of -2 are ± 1 , ± 2 .

$$\begin{aligned} p(1) &= 2 \times 1^3 + 1^2 - 2 \times 1 - 1 \\ &= 2 + 1 - 2 - 1 = 0. \end{aligned}$$

Therefore, $(y - 1)$ is a factor of $p(y)$.

$$\begin{aligned} \text{Now, } 2y^3 + y^2 - 2y - 1 &= 2y^3 - 2y^2 + 3y^2 - 3y + y - 1 \\ &= 2y^2(y - 1) + 3y(y - 1) + 1(y - 1) \\ &= (y - 1)(2y^2 + 3y + 1) \\ &= (y - 1)(2y^2 + 2y + y + 1) \\ &= (y - 1)\{2y(y + 1) + 1(y + 1)\} \\ &= (y - 1)(y + 1)(2y + 1) \end{aligned}$$

Therefore, $2y^3 + y^2 - 2y - 1 = (y - 1)(2y + 1)(y + 1)$. **Ans.**

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EXERCISE 2.5

Q.1. Use suitable identities to find the following products :

(i) $(x + 4)(x + 10)$ (ii) $(x + 8)(x - 10)$

(iii) $(3x + 4)(3x - 5)$ (iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$ (v) $(3 - 2x)(3 + 2x)$

- Sol.**
- (i) Using identity $(x + a)(x + b) = x^2 + (a + b)x + ab$
 $(x + 4)(x + 10) = x^2 + (4 + 10)x + 4 \times 10 = x^2 + 14x + 40$ **Ans.**
- (ii) Using the same identity as in (i) above
 $(x + 8)(x - 10) = x^2 + (8 - 10)x + 8 \times (-10) = x^2 - 2x - 80$ **Ans.**
- (iii) Using the same identity
 $(3x + 4)(3x - 5) = 3x \times 3x + (-1)(3x) - 20 = 9x^2 - 3x - 20$ **Ans.**
- (iv) Using $(x + y)(x - y) = x^2 - y^2$
 $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = (y^2)^2 - \left(\frac{3}{2}\right)^2 = y^4 - \frac{9}{4}$ **Ans.**
- (v) Using the same identity as in (iv)
 $(3 - 2x)(3 + 2x) = 3^2 - (2x)^2$
 $= 9 - 4x^2$ **Ans.**

Q.2. Evaluate the following products without multiplying directly :

(i) 103×107 (ii) 95×96 (iii) 104×96

- Sol.**
- (i) $103 \times 107 = (100 + 3)(100 + 7)$
 $= (100)^2 + (3 + 7) \times 100 + 3 \times 7$
 $= 10000 + 1000 + 21 = 11021$ **Ans.**
- (ii) $95 \times 96 = (100 - 5)(100 - 4)$
 $= (100)^2 - (5 + 4) \times 100 + 5 \times 4$
 $= 10000 - 900 + 20 = 9120$ **Ans.**
- (iii) $104 \times 96 = (100 + 4)(100 - 4) = 100^2 - 4^2$
 $= 10000 - 16 = 9984$ **Ans.**

Q.3. Factorise the following using appropriate identities :

$$(i) 9x^2 + 6xy + y^2 \quad (ii) 4y^2 - 4y + 1 \quad (iii) x^2 - \frac{y^2}{100}$$

Sol. (i) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)y + (y)^2$
 $= (3x + y)^2$ [Using $a^2 + 2ab + b^2 = (a + b)^2$]

$$= (3x + y)(3x + y) \quad \mathbf{Ans.}$$

(ii) $4y^2 - 4y + 1$
 $= (2y)^2 - 2(2y)(1) + (1)^2$
 $= (2y - 1)^2 = (2y - 1)(2y - 1)$ [Using $a^2 - 2ab + b^2 = (a - b)^2$] **Ans.**

(iii) $x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{10}\right)^2$
 $= \left(\frac{x + y}{10}\right)\left(\frac{x - y}{10}\right)$ [Using $a^2 - b^2 = (a + b)(a - b)$] **Ans.**

Q.4. Expand each of the following, using suitable identities :

$$(i) (x + 2y + 4z)^2 \quad (ii) (2x - y + z)^2 \quad (iii) (-2x + 3y + 2z)^2$$

$$(iv) (3a - 7b - c)^2 \quad (v) (-2x + 5y - 3z)^2 \quad (vi) \left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$$

Sol. (i) $(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2x \times 2y + 2 \times 2y \times 4z + 2 \times 4z \times x$
 $= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$ **Ans.**

(ii) $(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2 \times (2x)(-y)$
 $+ 2(-y)(z) + 2(z) \times 2x$
 $= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$ **Ans.**

(iii) $(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y)$
 $+ 2(3y)(2z) + 2(2z)(-2x)$
 $= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$ **Ans.**

(iv) $(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b)$
 $+ 2(-7b)(-c) + 2(-c)(3a)$
 $= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$ **Ans.**

(v) $(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y)$
 $+ 2(5y)(-3z) + 2(-3z)(-2x)$
 $= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$ **Ans.**

$$\begin{aligned}
 \text{(vi)} \quad \left[\frac{1}{4}a - \frac{1}{2}b + 1 \right]^2 &= \left(\frac{1}{4}a \right)^2 + \left(\frac{-1}{2}b \right)^2 + (1)^2 + 2 \left(\frac{1}{4}a \right) \left(\frac{-1}{2}b \right) \\
 &\quad + 2 \left(\frac{-1}{2}b \right) \times 1 + 2(1) \times \frac{1}{4}a \\
 &= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a \\
 &= \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2} \quad \text{Ans.}
 \end{aligned}$$

Q.5. Factorise :

$$(i) \quad 4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$$

$$(ii) \quad 2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$$

Sol.

$$\begin{aligned}
 (i) \quad 4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz \\
 &= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x) \\
 &= (2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z) \quad \text{Ans.}
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad 2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz \\
 &= (\sqrt{2}x)^2 + (-y)^2 + (-2\sqrt{2}z)^2 + 2(\sqrt{2}x)(-y) + 2(-y)(-2\sqrt{2}z) \\
 &\quad + 2(\sqrt{2}x)(-2\sqrt{2}z) \\
 &= (\sqrt{2}x - y - 2\sqrt{2}z)^2 \\
 &= (\sqrt{2}x - y - 2\sqrt{2}z)(\sqrt{2}x - y - 2\sqrt{2}z) \quad \text{Ans.}
 \end{aligned}$$

Q.6. Write the following cubes in expanded form :

$$(i) \quad (2x + 1)^3 \quad (ii) \quad (2a - 3b)^3 \quad (iii) \quad \left[\frac{3}{2}x + 1 \right]^3 \quad (iv) \quad \left[x - \frac{2}{3}y \right]^3$$

Sol.

$$\begin{aligned}
 (i) \quad (2x + 1)^3 &= (2x)^3 + 1^3 + 3(2x)(1)(2x + 1) \\
 &= 8x^3 + 1 + 6x(2x + 1) = 8x^3 + 12x^2 + 6x + 1 \quad \text{Ans.} \\
 (ii) \quad (2a - 3b)^3 &= (2a)^3 - (3b)^3 - 3 \times 2a \times 3b(2a - 3b) \\
 &= 8a^3 - 27b^3 - 18ab(2a - 3b) \\
 &= 8a^3 - 27b^3 - 36a^2b + 54ab^2 \quad \text{Ans.}
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad \left[\frac{3}{2}x + 1 \right]^3 &= \left(\frac{3}{2}x \right)^3 + 1^3 + 3 \left(\frac{3}{2}x \right) (1) \left(\frac{3}{2}x + 1 \right) \\
 &= \frac{27}{8}x^3 + 1 + \frac{9}{2}x \left(\frac{3}{2}x + 1 \right) \\
 &= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1 \quad \text{Ans.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \left[x - \frac{2}{3}y \right]^3 &= x^3 - \left(\frac{2}{3}y \right)^3 - 3(x) \left(\frac{2}{3}y \right) \left(x - \frac{2}{3}y \right) \\
 &= x^3 - \frac{8}{27}y^3 - 2xy \left(x - \frac{2}{3}y \right) \\
 &= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2 \quad \text{Ans.}
 \end{aligned}$$

Q.7. Evaluate the following using suitable identities :

$$\text{(i) } (99)^3 \qquad \text{(ii) } (102)^3 \qquad \text{(iii) } (998)^3$$

Sol. (i) $(99)^3 = (100 - 1)^3 = (100)^3 + (-1)^3 + 3(100)(-1)(100 - 1)$
 $= 1000000 - 1 - 300(100 - 1)$
 $= 1000000 - 1 - 30000 + 300 = 970299$

(ii) $(102)^3 = (100 + 2)^3 = 100^3 + 2^3 + 3(100)(2)(100 + 2)$
 $= 1000000 + 8 + 600(100 + 2)$
 $= 1000000 + 8 + 60000 + 1200 = 1061208 \quad \text{Ans.}$

(iii) $(998)^3 = (1000 - 2)^3 = (1000)^3 + (-2)^3 + 3(1000)(-2)(998)$
 $= (1000)^3 - 8 - 6000(998)$
 $= 1000000000 - 8 - 5988000 = 994011992 \quad \text{Ans.}$

Q.8. Factorise each of the following :

$$\begin{aligned}
 \text{(i)} \quad &8a^3 + b^3 + 12a^2b + 6ab^2 & \text{(ii)} \quad &8a^3 - b^3 - 12a^2b + 6ab^2 \\
 \text{(iii)} \quad &27 - 125a^3 - 135a + 225a^2 \\
 \text{(iv)} \quad &64a^3 - 27b^3 - 144a^2b + 108ab^2
 \end{aligned}$$

$$\text{(v)} \quad 27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$$

Sol. (i) $8a^3 + b^3 + 12a^2b + 6ab^2$
 $= (2a)^3 + b^3 + 3(2a)(b)(2a + b)$
 $= (2a + b)^3 = (2a + b)(2a + b)(2a + b) \quad \text{Ans.}$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$
 $= (2a)^3 + (-b)^3 + 3(2a)(-b)(2a - b)$
 $= (2a - b)^3 = (2a - b)(2a - b)(2a - b) \quad \text{Ans.}$

(iii) $27 - 125a^3 - 135a + 225a^2$
 $= 3^3 + (-5a)^3 + 3 \times (3)(-5a)(3 - 5a)$
 $= (3 - 5a)^3 = (3 - 5a)(3 - 5a)(3 - 5a) \quad \text{Ans.}$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$
 $= (4a)^3 + (-3b)^3 + 3(4a) \times (-3b)(4a - 3b)$
 $= (4a - 3b)^3 = (4a - 3b)(4a - 3b)(4a - 3b) \quad \text{Ans.}$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$
 $= (3p)^3 + \left(-\frac{1}{6} \right)^3 + 3(3p) \left(-\frac{1}{6} \right) \left(3p - \frac{1}{6} \right)$
 $= \left(3p - \frac{1}{6} \right)^3 = \left(3p - \frac{1}{6} \right) \left(3p - \frac{1}{6} \right) \left(3p - \frac{1}{6} \right) \quad \text{Ans.}$

Q.9. Verify : (i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Sol. (i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

R.H.S. = $x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

= $x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3 = x^3 + y^3 = \text{L.H.S.}$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

R.H.S. = $x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

= $x^3 + x^2y + xy^2 - yx^2 - xy^2 - y^3 = x^3 - y^3 = \text{L.H.S.}$

Q.10. Factorise each of the following :

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Sol. (i) $27y^3 + 125z^3 = (3y)^3 + (5z)^3 = (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$
 = $(3y + 5z)(9y^2 - 15yz + 25z^2)$ **Ans.**

(ii) $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

= $(4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$

= $(4m - 7n)(16m^2 + 28mn + 49n^2)$

Q.11. Factorise : $27x^3 + y^3 + z^3 - 9xyz$

Sol. $27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)yz$

= $(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$ **Ans.**

Q.12. Verify that :

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

Sol. To verify :

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

$$\text{R.H.S.} = \frac{1}{2}(x + y + z)[x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + z^2 + x^2 - 2zx]$$

$$= \frac{1}{2}(x + y + z)[2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx]$$

$$= (x + y + z)[x^2 + y^2 + z^2 - xy - yz - zx]$$

$$= x[x^2 + y^2 + z^2 - xy - yz - zx]$$

$$+ y(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$+ z(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= x^3 + xy^2 + xz^2 - x^2y - xyz - zx^2 + yx^2 + y^3 + yz^2 - xy^2$$

$$- y^2z - zxy + zx^2 + zy^2 + z^3 - zxy - yz^2 - z^2x$$

$$= x^3 + y^3 + z^3 - 3xyz = \text{L.H.S.} \quad \text{Hence verified.}$$

Q.13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Sol. $x + y + z = 0$

$$(x + y + z)^3 = x^3 + y^3 + z^3 - 3xyz = 0$$

$$\Rightarrow x^3 + y^3 + z^3 = 3xyz. \quad \text{Proved.}$$

Q.14. Without actually calculating the cubes, find the value of each of the following :

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Sol. From the above question, we have $x^3 + y^3 + z^3 = 3xyz$, if $x + y + z = 0$

(i) Here $-12 + 7 + 5 = 0$

$$(-12)^3 + (7)^3 + (5)^3$$

$$3(-12)(7)(5) = -1260 \quad \text{Ans.}$$

$$(ii) \text{ Here } 28 + (-15) + (-13) = 0$$

$$\begin{aligned} \text{So, } & (28)^3 + (-15)^3 + (-13)^3 \\ & = 3 \times 28 (-15) (-13) = 16380 \quad \mathbf{Ans.} \end{aligned}$$

Q.15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given :

$$\boxed{\text{Area : } 25a^2 - 35a + 12}$$

(i)

$$\boxed{\text{Area : } 35y^2 + 13y - 12}$$

(ii)

Sol. (i)
$$\begin{aligned} \text{Area} &= 25a^2 - 35a + 12 \\ &= 25a^2 - 20a - 15a + 12 \\ &= 5a(5a - 4) - 3(5a - 4) \\ &= (5a - 4)(5a - 3) \end{aligned}$$

So, one possible answer is length = $(5a - 4)$, breadth = $(5a - 3)$

Therefore $p\left(\frac{3}{5}\right)$ gives zero value and $(5a - 3)$ is a factor.

Second factor $(5a - 4)$, length = $(5a - 3)$; breadth = $(5a - 4)$.

(ii)
$$\begin{aligned} \text{Area} &= 35y^2 + 13y - 12 \\ &= 35y^2 + 28y - 15y - 12 \\ &= 7y(5y + 4) - 3(5y + 4) \\ &= (5y + 4)(7y - 3) \end{aligned}$$

So, $(5y + 4)$ may be taken as breadth and $(7y - 3)$ as length. **Ans.**

Q.16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

$$\boxed{\text{Volume : } 3x^2 - 12x}$$

(i)

$$\boxed{\text{Volume : } 12ky^2 + 8ky - 20k}$$

(ii)

Sol. (i) $abc = 3x^2 - 12x = 3x(x - 4)$

$3, x(x - 4)$ are the three factors so they can be three dimensions.

(ii)
$$\begin{aligned} abc &= 12ky^2 + 8ky - 20k \\ &= 4k(3y^2 + 2y - 5) \\ &= 4k\{3y(y - 1) + 5(y - 1)\} \\ &= 4k(y - 1)(3y + 5) \end{aligned}$$

$4k, (y - 1)$ and $(3y + 5)$ are the three factors, so they can be three dimensions **Ans.**